where $\bar{y}_F = y_F/y_0$, $\bar{x} = x/y_0$, and $N = \sigma B^2 V_0/\rho g$. Equation (5) is presented graphically in Fig. 2 for various values of N. It is evident that $\bar{y}_F$ goes to zero as $2N\bar{x}$ approaches unity, which is physically untenable. Indeed, for small values of $\bar{y}_F$, the inertia terms cannot be neglected, and Eq. (5) becomes invalid.

Results

The experimental results were obtained photographically, using the flow system shown schematically in Fig. 1. Figure 3 shows the experimentally determined coordinates of the free surface superimposed on the theoretical curves for corresponding values of N. The agreement is good, except near the exit, for curves characterized by large values of N. This discrepancy is probably the result of secondary flows in the form of strong corner vortices observed in the neighborhood of the exit.

Conclusions

The techniques that were successfully employed to solve the foregoing problem could be used to simplify other similar but more difficult magnetohydrodynamics (MHD) problems. For example, the fact that the x component of velocity is uniform with y is also true for the problem of the unsteady emptying of a fluid from a tank or the extension of the foregoing problem to cylindrical geometry.

Mass Transfer Cooling in Laminar Hypersonic Cavity Flow

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Nomenclature

D	=	cavity depth
H	=	local heat-transfer coefficient
$\bar{H}$	=	average heat-transfer coefficient
L	=	cavity length
M	=	Mach number
$\dot{M}$	=	gas injection rate
Q	=	local heat-transfer rate
$\bar{Q}$	=	average heat-transfer rate
Re_∞	=	freestream Reynolds number
X	=	distance measured from upstream cavity wall

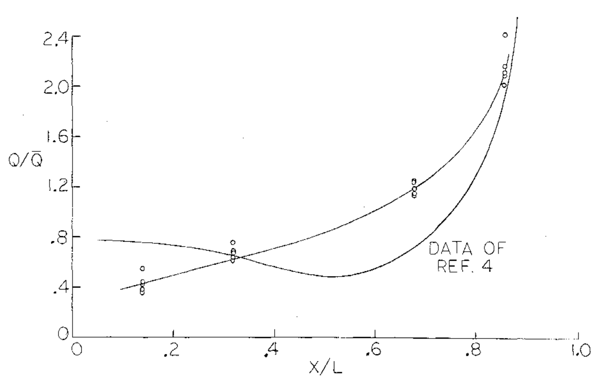

Fig. 1 Heat-transfer distribution with no injection.

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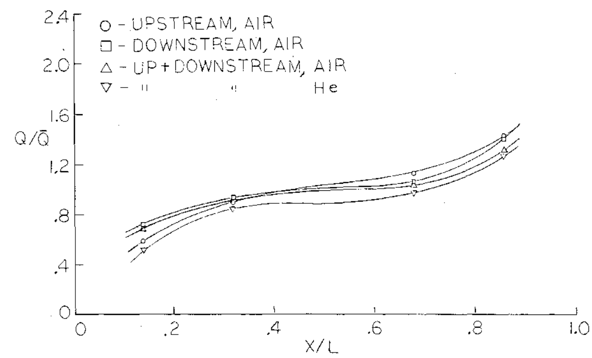

Fig. 2 Heat-transfer distribution with gas injection.

Analysis[1] has indicated that the heat-transfer rate through a separated laminar boundary layer would be reduced to 0.56 of that which would occur through the corresponding attached laminar boundary layer. This has been confirmed by Charwat, Dewey, et al.[2] who determined experimentally the heat-transfer distribution existing on the floor of a two-dimensional rectangular cavity in a separated supersonic stream. They found that, when the boundary-layer thickness at the upstream edge of the cavity was small in comparison to the depth of the cavity, the average heat-transfer rate to the floor of the cavity was reduced by a factor of approximately 2. Furey[3] obtained similar experimental results. Heat transfer in axisymmetric separated regions was experimentally determined by Larson[4] and Nicoll,[5] and again agreement with Ref. 1 was obtained.

Chapman[1] further predicted that the introduction of a coolant gas into a separated region would have a powerful effect in reducing the rate of heat transfer through the region. In fact, he predicted that a moderate quantity of gas injected into the separated region would reduce the heat transfer to zero. Presented here are experimental results showing the accuracy of this prediction.

Heat transfer with gas injection was measured for a rectangular cavity on a 20° total angle cone at a Mach number of 6.86 in air. The cavity flow was laminar (Re_∞/ft $\doteq 2 \times 10^6$) and "open" ($L/D = 6.7$). All experiments were conducted in the 3-in. hypersonic wind tunnel of IIT Research Institute which is described in Ref. 6.

Heat-transfer measurements were made using a transient calorimeter technique. The cavity floor was instrumented with five equally spaced copper-ring calorimeters. Means for injecting gas into the cavity were provided by two manifolds, which formed the upstream and downstream walls of the cavity.

In Fig. 1, the results are shown for the heat-transfer distribution in the cavity with no injection, which are compared to that obtained by Larson[4] for a rectangular cavity on a cylinder at $M = 4$. The average heat-transfer rate was found to be 50% of the heat-transfer rate of a corresponding

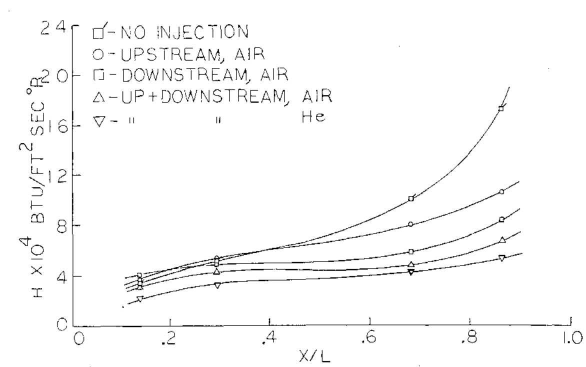

Fig. 3 Distribution of heat-transfer coefficient.

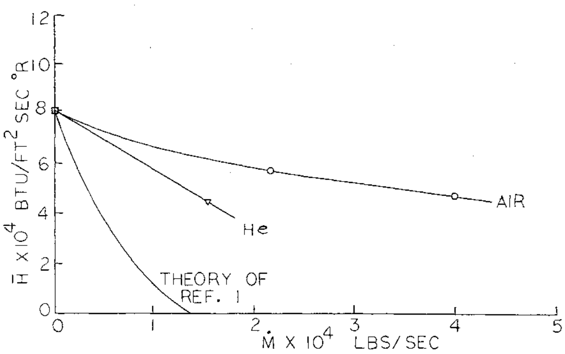

Fig. 4 Reduction in heat-transfer coefficient with injection.

attached boundary layer calculated by the method of Ref. 7. This result agrees with the predictions of Chapman.

The heat-transfer distributions with gas injection are shown in Fig. 2. Data were obtained for four cases: 1) air injection from the upstream cavity wall ($\dot{M} = 2.26 \times 10^{-4}$ lb/sec), 2) air injection from the downstream cavity wall, ($\dot{M} = 2.15 \times 10^{-4}$ lb/sec), 3) air injection from both walls simultaneously ($\dot{M} = 4.00 \times 10^{-4}$ lb/sec), and 4) helium injection from both walls simultaneously ($\dot{M} = 1.56 \times 10^{-4}$ lb/sec). These results indicate that gas injection into a region of laminar separation reduces the large heat-transfer rate that occurs in the downstream reattachment region and yields a more uniform heat-transfer distribution. In addition, the manner in which the coolant gas is introduced into the cavity (i.e., upstream, downstream, etc.) has little influence on the heat-transfer distribution.

In Fig. 3, the results are shown for the convective heat-transfer coefficient based upon a recovery factor equal to the square root of the Prandtl number. This has been suggested as a reasonable recovery factor for laminar-separated flow without gas injection.[1, 5] It is clear that helium is a much more effective coolant than air. The greatest reductions in heat transfer occurred in the reattachment region of the cavity with little if any reductions in the upstream portions of the cavity.

In Fig. 4, the reduction of the average heat-transfer coefficient with injection of air or helium is shown. These results are compared to the predictions of Chapman[1] for the case $M = \infty$. The reductions in heat transfer attained in the present study with air and helium as coolants, though substantial, are not as great as predicted by theory. Whereas the theory of Ref. 1 considers only air injection, the present study indicates that helium is a more effective coolant than air.

The following comments and conclusions can be made from the preceding results:

1) The heat transfer to a region of laminar separated flow can be substantially reduced by gas injection.

2) Chapman's theory appears to overestimate the effectiveness of mass injection.

3) The greatest reductions in heat transfer occur in the regions where heat transfer is greatest. This results in more uniform distributions of heat transfer in the cavity.

4) The injection of a lightweight gas of high specific heat, such as helium, leads to greater reductions in the heat transfer than does the same quantity of air injection.

5) The position of the gas injection had little influence upon the heat-transfer distribution that was obtained.

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An Estimate of the Decay of Turbulent Intensity in a Shear Flow

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Introduction

A CONSIDERABLE effort is being expended currently to obtain estimates of the radar cross section of ionized turbulent wakes. The simplest case prevails when the plasma is very tenuous, the radar illuminating frequency is much larger than the plasma frequency of the ionized gas, and the effects of multiple scattering can be ignored. When all of these conditions are met, the radar cross section can be calculated by the first Born approximation. The calculated cross section is proportional to the mean value of the square of the electron-density fluctuations and the three-dimensional Fourier transform of the correlation function (spectrum) of the effective dielectric constant of the plasma.

A previous paper[1] predicted the radar cross section of a turbulent underdense wake based on the following three assumptions: 1) the spectrum of the fluctuations of the index of refraction of the plasma was assumed to be proportional to the turbulent energy spectrum, 2) a Kolmogoroff spectrum was assumed to prevail, and 3) the turbulent energy was assumed constant. In this paper the assumption of constant energy is examined in some detail.

Analysis

The turbulent energy for constant density is given by the temporal mean, which we assume to be equal to the ensemble average

$$\langle E \rangle = \tfrac{1}{2} \langle u^2 + v^2 + w^2 \rangle$$

where, for convenience, we have set the density $\rho \equiv 1$, and u, v, and w are the components of the turbulent velocity fluctuations. Since we shall deal only with mean values, we shall drop the angular brackets, it being understood that temporal mean values are to be taken.

The turbulent energy E is assumed here to be a known function of the wave number κ. In the development that follows, we assume that, throughout the process of decay, the energy spectrum $E(\kappa)$ remains self-similar, and only the amplitude decreases uniformly over all of the frequencies.

The time rate of change of the energy in a flowing medium is given by the substantial derivative

$$DE/Dt \equiv (\partial E/\partial t) + \bar{q} \cdot \nabla E$$

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